ISSN 2229-5518

# Hall Effects on Steady MHD Boundary Layer Flow and Heat Transfer Due to Stretching Plate in the Presence of Heat Source or Sink

Mohammad Ali ,Mohammad Shah Alam, Asaduzzaman

**Abstract**— In the present paper is an investigation of heat and mass transfer of a steady flow of an incompressible electrically conducting fluid due to stretching plate under the influence of an applied uniform magnetic field and the effects of Hall current are taken into account. Where the flow is generated due to a linear stretching plate. Using suitable similarity transformations the governing equations of the problem are reduced to couple nonlinear ordinary differential equations and are solved numerically by Runge- Kutta fourth-fifth order method using symbolic software MATLAB. The numerical results concerned with the velocity, secondary velocity, and temperature profiles effects of various parameters on the flow fields are investigated and presented graphically. The results have possible technological applications in liquid-based systems involving stretchable materials.

**Index Terms**— Boundary layer, Hall current, Heat Source/Sink ,Heat and Mass transfer, MHD, Steady, Stretching plate.

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| **MHD** | Magnetohydrodynamics |
| $c_p$ | Specific heat of with constant pressure |
| g | Non-dimensional Secondary Velocity |
| $f'$ | Velocity Profile |
| M | Magnetic parameter, $M = \frac{\sigma B_0^2}{\rho a}$ |
| m | Hall parameter |
| ν | Kinematic viscosity |
| η | Similarity variable |
| $\alpha$ | Thermal diffusivity |
| $\rho$ | Density |
| σ | Fluid electrical conductivity |
| θ | Dimensionless temperature |
| u | Velocity component in x-direction |
| v | Velocity component in y-direction |
| w | Secondary Velocity |
| T | Temperature |
| $P_r$ | Prandle number, $P_r = \frac{\nu}{\alpha}$ |
| $B_0$ | Constant magnetic field intensity |
| $T_w$ | Temperature at the Plate |
| $T_\infty$ | Temperature of the fluid outside the boundary layer |
| δ | Dimensionless source/sink parameter |
| **Subscripts** | |
| W | Quantities at wall |
| ∞ | Quantities at the free stream |

## 1 Introduction

The study of boundary layer flow and heat transfer due to a stretching plate in the presence of heat source has generated much interest in recent years in view of its significant applications in industrial manufacturing processes such as glass-fiber, aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, which would be in the form of an electrolyte, and polymer sheet extruded continuously from a die are few practical applications of moving surfaces. Glass blowing, continuous casting, paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion, metal spinning and spinning of fibers also involve the flow due to stretching surface.

During its manufacturing process a stretched sheet interacts with the ambient fluid thermally and mechanically. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari& Keller [1]).Recently,A.K. Banerjee et al. Study the unsteady MHD flow past a stretching sheet due to a source/sink, Liu [2] studied heat and mass transfer in a MHD flow past a stretching sheet including the chemically reactive species of order one and internal heat generation or absorption. Xu[3] studied the free convective heat transfer characteristics in an electrically conducting fluid near an isothermal sheet with internal heat generation or absorption. Abel and Mahesha[4] was studied viscous dissipation effect along with heat transfer in MHD viscoelastic fluid flow over a stretching sheet. Their study also includes the effect of variable thermal conductivity, non-uniform heat source and radiation. Khan [5] studied the effect of heat transfer on a viscoelastic fluid flow over a stretching sheet with heat source/sink, suction/blowing and radiation. Pal and Talukdar[6] studied the unsteady MHD heat and mass transfer along with heat source past avertical permeable plate using a perturbation analysis, where the unsteadiness is

ISSN 2229-5518

caused by the time dependent surface temperature and concentration. Unsteady flow and heat transfer over an unsteady stretching sheet was studied by Ishak et al. [7]. Liu and Andersson[8] have also studied the heat and flow transfer over an unsteady stretching sheet. Unsteady flow and heat transfer with viscous dissipation of a non-Newtonian fluid over an unsteady stretching sheet was considered by Chen [9].Chebyshev finite difference method was used by Tsai et al. [10] to study the unsteady flow and heat transfer over an unsteady stretching surface with non-uniform heat source. Mukhopadhyay[11] studied the unsteady heat and flow transfer along with radiation effect over an unsteady stretching sheet. It must be noted that the unsteadiness in Ishak et al. [7]-Tsai et al. [10] are due to time-dependent stretching rate and temperature of the sheet.

In the present paper, the transient changes in the temperature field is due to the heat source/sink on the steady MHD boundary layer flow and heat transfer past a linearly stretching sheet. The effects of heat source/sink, magnetic field, Hall parameter and Prandtl number on the temperature field and on the velocity profiles are analysed.

## 2 Mathematical formulation of the Problem

Let us consider steady two dimensional laminar boundary layer over a stretching sheet immersed in an incompressible electrically conducting fluid with heat source or sink. The flow is subjected to a transverse magnetic field of strength $B_0$ which is assumed to be applied in the positive y –direction, normal to the surface. The induced magnetic field is assumed to be small compared to the applied magnetic field and is neglected. The flow is generated due to stretching sheet, caused by simultaneous application of two equal and opposite forces, so that the sheet is stretched keeping the origin fixed



Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, and energy under the influence of externally imposed magnetic field are:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma {B_0}^2}{\rho\left(1+m^2\right)}\left(u+mw\right) \qquad (2)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\nu\frac{\partial^2 w}{\partial y^2}+\frac{\sigma {B_0}^2}{\rho\left(1+m^2\right)}\left(mu-w\right) \qquad (3)$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c_p}\left(T-T_\infty\right) \qquad (4)$$

Boundary conditions are:

$$u=u_w\left(x\right), v=w=0, T=T_w \text{ at } y=0$$
$$u=0, w=0, T=T_\infty \text{ as } y\to\infty \qquad (5)$$
$$\text{where } u_w\left(x\right)=ax, \text{and } T_w=T_\infty+bx$$

In the governing equations x and y represent coordinate axes along the direction of motion and normal to it respectively. The velocity components along x and y -axis are u and v respectively. Heat source/sink denoted by Q. For heat source Q>0 and for sink Q<0.

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$w=axg\left(\eta\right), \eta=y\sqrt{\frac{a}{\nu}}, \quad \psi=x\sqrt{\left(a\nu\right)}f\left(\eta\right)$$
$$T-T_\infty=\theta\left(\eta\right)\left(T_w-T_\infty\right) \qquad (6)$$

The equation of continuity is satisfied if we choose a stream function $\psi\left(x,y\right)$ such that $u=\frac{\partial u}{\partial y}, \ v=-\frac{\partial v}{\partial x}$.

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained as

$$f^{'''}+ff^{''}-f^{'2}-\frac{M}{1+m^2}f^{'}-\frac{Mm}{1+m^2}g=0 \qquad (7)$$

$$g^{''}+g^{'}-f^{'}g+\frac{Mm}{1+m^2}f^{'}-\frac{M}{1+m^2}g=0 \qquad (8)$$

$$\theta^{''}+P_r f\theta^{'}-P_r f^{'}\theta+P_r\delta\theta=0 \qquad (9)$$

Where the notation primes denote differentiation with respect to η and the parameters are defined as

$$P_r=\frac{\nu}{\alpha}, \ M=\frac{\sigma B_0^2}{\rho a}, \ \alpha=\frac{K}{\rho c_p}, \ \delta=\frac{Qx}{\rho u_w c_p} \qquad (10)$$

The transform boundary conditions:

$$f=g=0, f^{'}=1, \theta=1 \text{at } \eta=0 \quad \text{and} \quad f=g=\theta=0 \text{ as } \eta\to\infty$$

## 3 Results and Discussion

Steady boundary layer flow and heat transfer of an incompressible electrically conducting fluid over a stretchings heet with source/sink in presence of Hall effect be examined. The boundary layer equations of momentum and heat transfer are solved numerically by Runge- Kutta fourth-fifth order method using symbolic software MATLAB. The temperature profiles of fluid are depicted graphically for various values of the parameters such as magneticparameter M, Hall parameter m, source/sink parameter δ and Prandtl number Pr.The Fig.1-Fig.4 describes the velocity profiles due to magnetic parameter M and Hall parameter m. It is observed that for various values

ISSN 2229-5518

of M the velocity profile increase in a certain interval then decrease (Fig.1& Fig.2). From Fig.3 and Fig.4 it is shown that the velocity profiles also decrease for various values of m. In Fig.5-Fig.8 shown the effect of M and m on secondary velocity profile.It is observed that the secondary velocity is decreased for various values of M but the velocity is increased for various values of m. The Fig.9- Fig.14 describes the temperature profiles due to heat source/heat sink and Prandlt number. From the Figs. 9 and 10 it is observed that the temperature profiles decreases for the heat source (Q > 0) but Figs. 13 and 14 shows that the temperature profiles increases for the heat sink (Q < 0) parameter . From Fig.11 and Fig.12it is observed that the temperature profiles decreases for various values of Pr

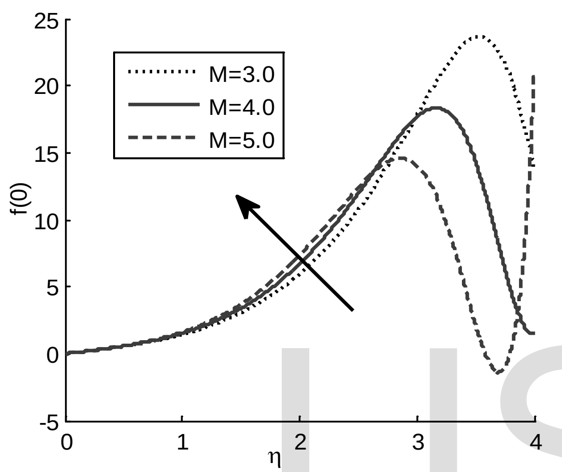


Fig. 1 Velocity profile for various values of M and m=1.0, $\delta$ =1.0, Pr=0.7

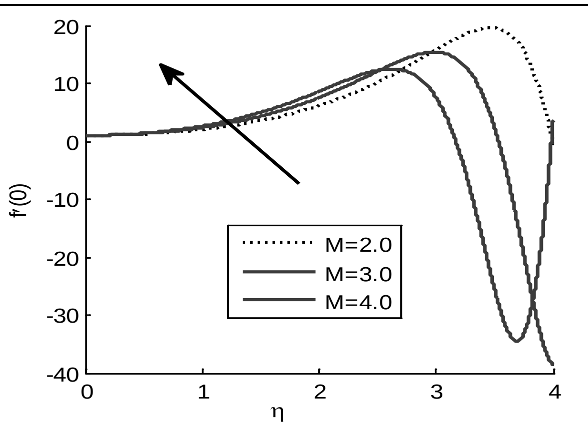


Fig. 2 Velocity profile for various values of M and m=1.0, $\delta$ =1.0, Pr=0.7

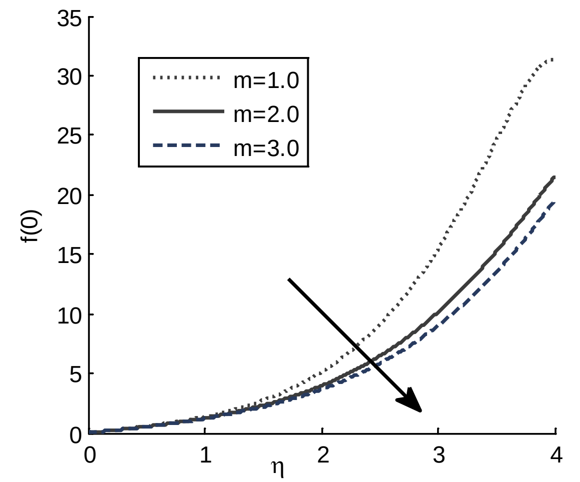


Fig. 3 Velocity profile for various values of m and M=2.0, $\delta$ =1.0, Pr=0.7

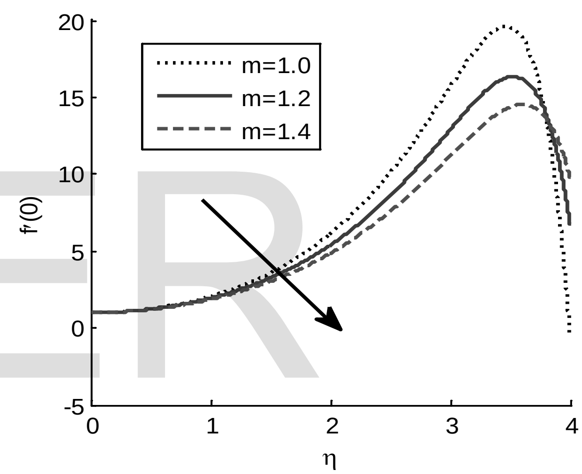


Fig. 4 Velocity profile for various values of m and M=2.0, $\delta$ =1.0, Pr=0.7

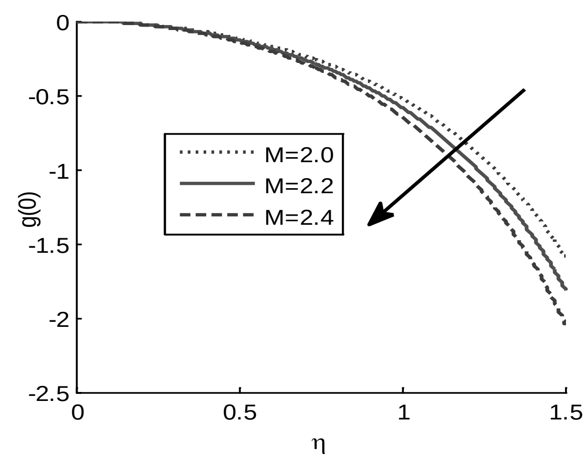


Fig. 5 Secondary velocity profile for various values of M and m=1.0. $\delta$ =1.0. Pr=0.7

ISSN 2229-5518

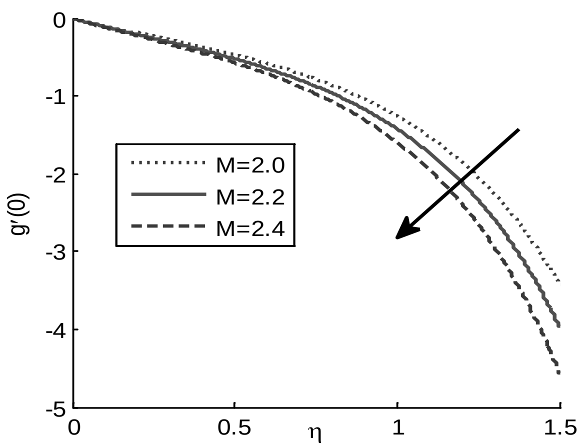


Fig. 6 Secondary velocity profile for various values of M and m=1.0, $\delta$ =1.0, Pr=0.7

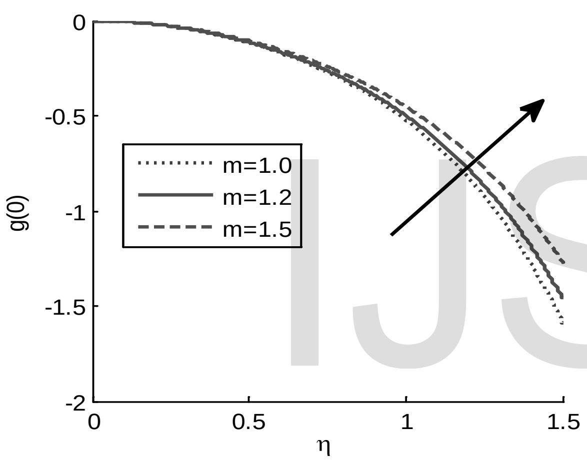


Fig. 7 Secondary velocity profile for various values of m and M=2.0, $\delta$ =1.0, Pr=0.7

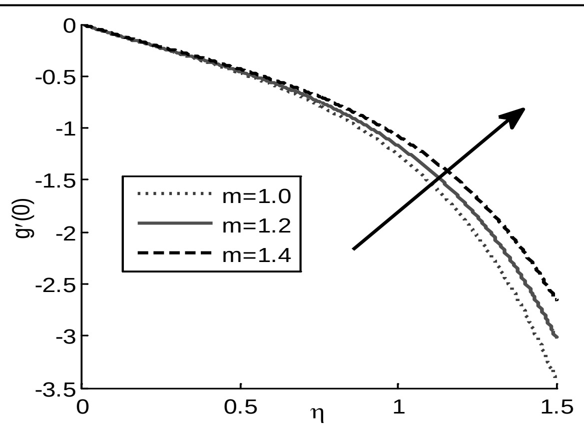


Fig. 8 Secondary velocity profile for various values of m and M=2.0, $\delta$ =1.0, Pr=0.7

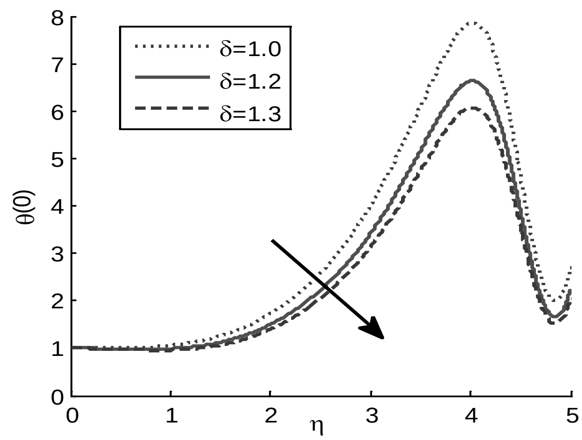


Fig. 9 Temperature profile for various values of $\delta$ and m=1.0, M = 2.0, Pr=0.7

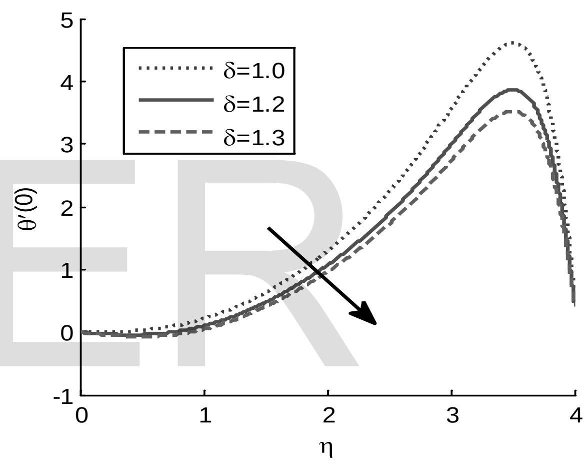


Fig. 10 Temperature profile for various values of $\delta$ and m=1.0, M = 2.0, Pr=0.7

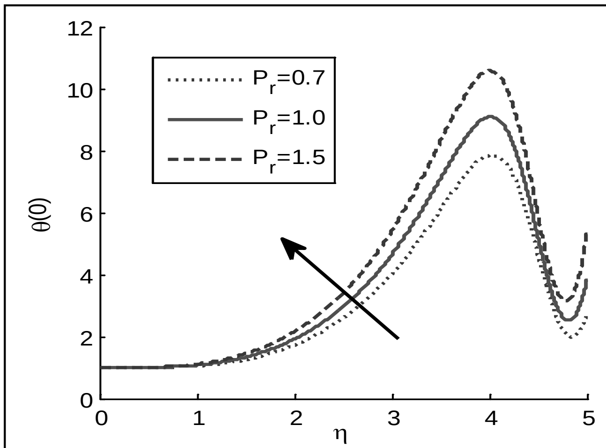


Fig. 11 Temperature profile for various values of Pr and m=1.0, M = 2.0, $\delta$ =1.0

ISSN 2229-5518

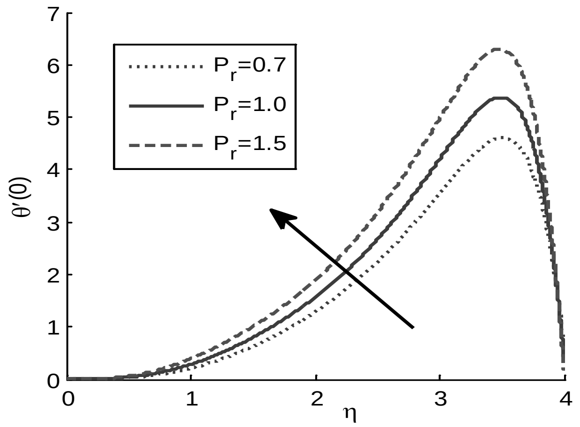


Fig. 12 Temperature profile for various values of Pr and m=1.0, M = 2.0, $\delta$ =1.0

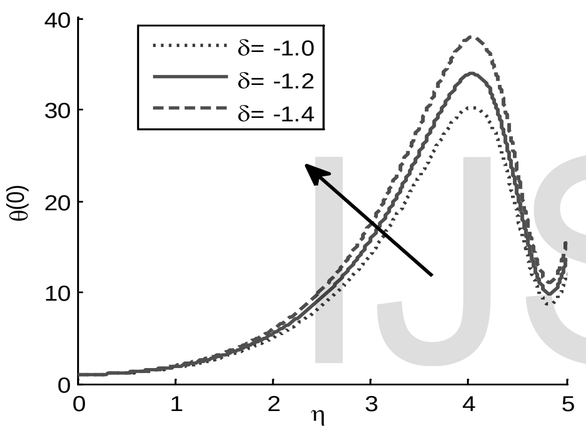


Fig. 13Temperature profile for various values of $\delta$ and m=1.0, M = 2.0, Pr=0.7

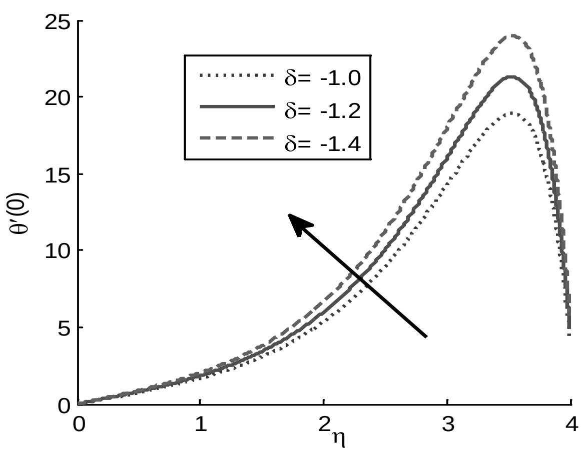


Fig. 14 Temperature profile for various values of $\delta$ and m=1.0, M = 2.0, Pr=0.7

## 4 Conclusion

The following important observations are made:

i. For the effects of magnetic field the velocity profile decreases in a certain interval of the value of η and then increase.

ii. For the effects of heat source the temperature decreasesbut the temperature increases in the case of the effect of heatsink.

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ISSN 2229-5518

Mohammad Ali
Assistant Professor
Department of Mathematics,
Chittagong University of Engineering and Technology (CUET),
Chittagong-4349, Bangladesh
Corresponding Email:ali.mehidi93@gmail.com

Mohammad Shah Alam
Assistant Professor
Department of Mathematics,
Chittagong University of Engineering and Technology (CUET),
Chittagong-4349, Bangladesh
Email:shahalammaths@gmail.com

Asaduzzaman
Assistant Professor
Department of Computer Science and Engineering
Chittagong University of Engineering and Technology (CUET),
Chittagong-4349, Bangladesh
Email: asad@cuet.ac.bd